

Simplify and Evaluate Algebraic Expressions



Getting the Idea

To simplify a numerical expression, follow the order of operations.

Order of Operations

1. Perform operations inside parentheses or other grouping symbols.
2. Evaluate exponents.
3. Multiply or divide in order from left to right.
4. Add or subtract in order from left to right.

Example 1

Simplify this expression.

$$\frac{1}{2}(2^3 + 2)$$

Strategy Follow the order of operations.

Step 1 Perform operations within parentheses.

The expression within parentheses is $(2^3 + 2)$.

Evaluate the exponent first, then add.

$$2^3 + 2 =$$

$$8 + 2 = 10$$

$$\text{So, } \frac{1}{2}(2^3 + 2) = \frac{1}{2}(10).$$

Step 2 Multiply.

$$\frac{1}{2}(10) = \frac{1}{2} \times 10 = 5$$

Solution $\frac{1}{2}(2^3 + 2) = 5$

You can use number properties and like terms to help you simplify algebraic expressions.

Like terms are terms that contain the same variable(s) raised to the same power(s).

Commutative Properties	
commutative property of addition $a + b = b + a$	commutative property of multiplication $ab = ba$

Associative Properties	
associative property of addition $(a + b) + c = a + (b + c)$	associative property of multiplication $(a \times b) \times c = a \times (b \times c)$

Example 2

Simplify this expression.

$$(11k + 5) + 2k$$

Strategy Use number properties and like terms.

Step 1 Use the commutative property to reorder the first two terms.

$$(11k + 5) + 2k =$$

$$(5 + 11k) + 2k$$

$11k$ and $2k$ are like terms. The like terms are next to each other.

Step 2 Use the associative property to group like terms.

$$(5 + 11k) + 2k =$$

$$5 + (11k + 2k)$$

Step 3 Combine the like terms.

$$5 + (11k + 2k) =$$

$$5 + 13k$$

Solution The expression can be simplified as $5 + 13k$.

Example 3

Simplify this expression.

$$4s + 5t + (-3s) + 4t$$

Strategy Use the properties of addition.

Step 1 Use the commutative property to reorder the terms.

$$4s + 5t + (-3s) + 4t =$$

$$4s + (-3s) + 5t + 4t$$

Step 2

Use the associative property to group like terms and combine them.

$$\begin{aligned} 4s + (-3s) + 5t + 4t &= \\ [4s + (-3s)] + (5t + 4t) &= \\ s + 9t & \end{aligned}$$

Solution The expression can be simplified to $s + 9t$.

To expand an expression is to remove parentheses or brackets.

You can use the distributive property to expand an expression.

Distributive Properties	
distributive property over addition	distributive property over subtraction
$a(b + c) = ab + ac$	$a(b - c) = ab - ac$

Example 4

Expand this expression.

$$2(4m + n) - 2n$$

Strategy

Use number properties and combine like terms.

Step 1

Expand the first part of the expression using the distributive property.

$$\begin{aligned} 2(4m + n) &= (2 \times 4m) + (2 \times n) \\ &= 8m + 2n \end{aligned}$$

Step 2

Rewrite the expression.

$$2(4m + n) - 2n = 8m + 2n - 2n$$

Step 3

Use the associative property to group and combine like terms.

$$\begin{aligned} 8m + (2n - 2n) &= \\ 8m + 0n &= 8m \end{aligned}$$

Solution

Expanded, the expression is $8m$.

The opposite of expanding is factoring. You can also use the distributive property to help you factor an expression. An expression is completely factored when there are no more common factors among terms.

Example 5

Simplify and factor this algebraic expression.

$$6x + 3x + 15y + 12y$$

Strategy **Combine like terms. Then use the distributive property to find the GCF.**

Step 1

Combine like terms.

$$\begin{aligned}6x + 3x + 15y + 12y &= \\9x + 27y &\end{aligned}$$

Step 2

Find the greatest common factor (GCF) of the terms $9x$ and $27y$.

The GCF of $9x$ and $27y$ is 9.

Step 3

Factor 9 from each term in $9x + 27y$.

$$\begin{aligned}9x + 27y &= \\9 \times x + 9 \times 3y &= 9(x + 3y)\end{aligned}$$

Solution **The simplified and factored expression is $9(x + 3y)$.**

To evaluate an algebraic expression, substitute the given values for the variables. Remember to follow the order of operations.

Example 6

Evaluate this algebraic expression when $a = 8$ and $b = -7$.

$$12 + 3a - b$$

Strategy **Substitute the value of each variable into the expression. Then evaluate.**

Step 1

Substitute 8 for a and -7 for b .

$$12 + 3a - b = 12 + 3(8) - (-7)$$

Step 2

Use the order of operations to simplify.

First, multiply and divide from left to right.

$$12 + 3(8) - (-7) = 12 + 24 - (-7)$$

Next, add and subtract from left to right.

$$\text{Add: } 12 + 24 - (-7) = 36 - (-7) = 36 + 7 = 43$$

Solution **The value of the expression is 43.**

Some algebraic expressions contain **exponents**. Remember that an exponent tells you how many times a number is used as a factor.

Example 7

Evaluate this algebraic expression when $m = -3$ and $n = -4$.

$$m^2 + n$$

Strategy **Substitute the value of each variable into the expression. Then evaluate.**

Step 1 Substitute -3 for m and -4 for n .

$$m^2 + n = (-3)^2 + (-4)$$

Step 2 Use the order of operations to simplify.

First, evaluate the exponent.

$$(-3)^2 + (-4) =$$

$$(-3 \times -3) + (-4) =$$

$$9 + (-4)$$

Add.

$$9 + (-4) = 5$$

Solution **The value of the expression is 5.**



Coached Example

What is the value of this expression when $p = 8$ and $q = 5$?

$$\frac{16}{p} - 3q$$

Substitute _____ for p and _____ for q in the expression.

$$\frac{16}{p} - 3q = \underline{\hspace{4cm}}$$

Use the order of operations to simplify.

First, multiply and divide from left to right.

Now, add and subtract in order from left to right.

The value of the expression is _____.



Lesson Practice

Choose the correct answer.

1. Simplify the expression.

$$(8 - 5)^2 \cdot 3 - 8 \div 2$$

- A. -3
- B. 9.5
- C. 14
- D. 23

2. Simplify the expression.

$$\frac{28 + 4^2}{2}$$

- A. 9
- B. 11
- C. 16
- D. 22

3. Which is equivalent to the expression below?

$$4a + 5 - a + 2$$

- A. $10a$
- B. $11a$
- C. $3a + 7$
- D. $4a + 7$

4. Expand the expression.

$$6(2k - 3)$$

- A. $8k - 3$
- B. $9k$
- C. $12k - 3$
- D. $12k - 18$

5. What is the value of this expression when $c = 4$?

$$4c + 3c - 2c$$

- A. 20
- B. 28
- C. 36
- D. 40

6. What is the value of this expression when $a = -3$ and $b = 5$?

$$a + b^2$$

- A. -28
- B. -13
- C. 22
- D. 28

7. Which is equivalent to the expression below?

$$3p + 4 + p + 12 + 3q$$

- A. $4p + 3q + 16$
 B. $4p + 19q$
 C. $6p + q + 16$
 D. $20p + 3q$

8. Which shows the simplified and completely factored form of the following expression?

$$12x + 4x + 25y + 15y$$

- A. $2(8x + 10y)$
 B. $4(4x + 10y)$
 C. $5(3x + 5y)$
 D. $8(2x + 5y)$

9. Winnie wrote out the following expression.

$$8a - 2b + 12a - 2a - 4b$$

- A. Show how to simplify the expression. Write the answer in factored form.

- B. Evaluate the expression when $a = 4$ and $b = -2$. Show your work.

10. Draw a line from each expression to its value.

- | | | | |
|-------------------------------|---|---|----|
| A. $6^2 - 30 + 5$ | • | • | 1 |
| B. $(20 + 4 \times 9) \div 8$ | • | • | 7 |
| C. $10(7 - 2) + 20$ | • | • | 11 |
| D. $28 \div 7 - 3$ | • | • | 70 |

11. Which expression is equivalent to the one given below? Circle all that apply.

$$18r + (7 - 2) \times 3m$$

- A. $18r + 5 + 3m$
- B. $18r + 15m$
- C. $18r + m$
- D. $3(6r + 5m)$
- E. $15m + 18r$
- F. $23r + 3m$
- G. $m + 18r$

12. Evaluate the expression for each given value of x . Circle each correct value.

$$7x - 5(2x + 9)$$

$x = -3$ value of expression =

- 6
- 36
- 54

$x = 10$ value of expression =

- 21
- 39
- 75

$x = 7$ value of expression =

- 1,012
- 30
- 66

13. Look at each equation. Is the expression on the left simplified correctly on the right?
Select Yes or No.

A. $8p + 10q - 4p = 12p + 10q$ Yes No

B. $15p - 2(3q + p) = 13p - 6q$ Yes No

C. $\frac{10p}{2q} = \frac{5p}{q}$ Yes No

D. $-5p \times -3q = -15pq$ Yes No

14. Select True or False for each equation.

A. $-10 \times 3(6 - 4) = -60$ True False

B. $\frac{9}{6^2} = \frac{1}{4}$ True False

C. $15 - 6 \times -4 = -36$ True False

D. $7^2 + (10 - 5) = 54$ True False